Quantum Field Theory II FS24

Series 11 - The Renormalization Group. 29.05.2024

Part 1: Beta functions in Yukawa theory. The Lagrangian for the pseudoscalar Yukawa theory with masses set to zero takes the following form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4!}\phi^4 + \bar{\psi}(i\partial\!\!\!/)\psi - ig\bar{\psi}\gamma^5\psi\phi. \tag{1}$$

The goal of this exercise is to calculate the Callan-Synmanzik β functions for λ and g, i.e. $\beta_\lambda(\lambda, g)$, $\beta_g(\lambda, g)$ to leading order in coupling constants, assuming that λ and g^2 are of the same order.

In order to do that you have to first calculate the divergent part (i.e. the pole as $d \to 4$) of each counterterm at leading order in perturbation theory, implementing a sufficient set of renormalization conditions (you do not need to worry about the finite part of the counterterms, since they will not be needed for the β functions).

Hint: You have to calculate the divergent part of all the one-loop diagrams for the Yukawa theory. You have conducted similar calculations in many of the previous series.

Part 2: Asymptotic Symmetry.
(This is a rather long but very instructive exercise - consider it as optional.)
Consider the following Lagrangian with two scalar fields ϕ_1 and ϕ_2

$$\mathcal{L} = \frac{1}{2}((\partial_\mu \phi_1)^2 + (\partial_\mu \phi_2)^2) - \frac{\lambda}{4!}(\phi_1^4 + \phi_2^4) + \bar{\psi}(i\partial\!\!\!/)\psi - \frac{2\rho}{4!}\phi_1^2\phi_2^2. \tag{2}$$

(a) Is there any special symmetry for $\rho = \lambda$?

(b) Working in $d = 4$ dimensions, find the beta functions β_ρ and β_λ to leading order in the coupling constants.

Hint: Proceed in the same way as in Part 1.

(c) Write the renormalization group equation for the ratio of couplings ρ/λ and find the fixed points of the Renormalization Group (RG) flow.

(d) How do the beta functions β_ρ and β_λ change when working in $d = 4 - \epsilon$ dimensions ? Do the fixed points you calculated in (c) change in this case and why ? Draw the diagram of the RG flow in the ρ-λ plane for $\epsilon = 0.01$.

Hint: You can do the plot on your laptop using Mathematica or any other programming language you prefer.